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The Lifshitz Tail and Relaxation to Equilibrium in the One-Dimensional Disordered Ising Model

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We study spectral properties of the generator of the Glauber dynamics for a 1D disordered stochastic Ising model with random bounded couplings. By an explicit representation for the upper branch of the generator we get an asymptotic formula for the integrated density of states of the generator near the upper edge of the spectrum. This asymptotic behavior appears to have the form of the Lifshitz tail, which is typical for random operators near fluctuation boundaries. As a consequence we find the asymptotics for the average over the disorder of the time-autocorrelation function to be

$$\langle \langle \sigma_0^{\omega}(t), \sigma_0(0) \rangle_{\mathscr{P}(\omega)} \rangle_{\omega} = \exp\{-gt - kt^{1/3}(1+o(1))\} \qquad \text{as} \quad t \to \infty$$

with constants g, k depending on the distribution of the random couplings.

KEY WORDS: Glauber dynamics; stochastic Ising model; stochastic disordered systems; density of states; asymptotics of decay of correlations.

1. INTRODUCTION AND MAIN RESULTS

We consider the Glauber dynamics for a one-dimensional random Ising model with formal Hamiltonian

$$H(\sigma, \omega) = \sum_{\substack{n, n' \in \mathbf{Z} \\ |n-n'|=1}} \omega_{n, n'} \sigma_n \sigma_{n'}, \qquad \sigma_n = \pm 1 \tag{1}$$

Here $\sigma \in \Omega = \{+1, -1\}^{\mathbf{Z}}$; $\omega_{n, n'} = \omega_{n', n} = \omega_b \in \mathbf{R}$ are independent identically distributed random variables marked by bonds $b = (n, n')$, $|n-n'| = 1$ of

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the lattice $\mathbf{Z}$, and we denote by $p_b = p$ the common probability distribution of ω_b. Let $\mathbf{B}$ be the set of all bonds of $\mathbf{Z}$. Then the family of random variables $\omega = \{\omega_b, b \in \mathbf{B}\}$ forms a random field on $\mathbf{B}$ with the space of realizations $\mathbf{R}^{\mathbf{B}}$ and the probability distribution $\mathbf{P} = p^{\mathbf{B}}$. The random field ω is ergodic with respect to the group of translations on the lattice $\mathbf{Z}$.

It is known (see, e.g., ref. 1) that for any fixed realization of the random couplings ω and for any temperature $T = 1/\beta$ the random spin system with Hamiltonian (1) has a unique limit Gibbs measure $\mu_\beta(\omega)$. The measure $\mu_\beta(\omega)$ determines a non-homogeneous Markov chain on $\mathbf{Z}$ with the state space $\{-1, 1\}$, transition probabilities

$$P_\omega(\sigma_{n+1} \mid \sigma_n) = \frac{e^{\beta\omega_{n,n+1}\sigma_n\sigma_{n+1}}}{2\cosh \beta\omega_{n,n+1}}, \qquad n \in \mathbf{Z}, \quad \sigma_{\mathbf{n}} = \pm\mathbf{1}$$

and the stationary distribution $\nu_0(\sigma = 1) = \nu_0(\sigma = -1) = \frac{1}{2}$ (see in details ref. 2).

We denote by $\mathscr{H}_\omega = \mathscr{L}_2(\Omega, d\mu_\beta(\omega))$ the space of functions on Ω, and we define the generator of the stochastic Ising model with Hamiltonian (1) by the following way:

$$L^\beta(\omega)\, f(\sigma) = \sum_{n \in \mathbf{Z}} c(n, \sigma, \omega)(f(\sigma^{(n)}) - f(\sigma)), \qquad f(\sigma) \in \mathscr{D} \subset \mathscr{H}_\omega \tag{2}$$

The operator (2) is defined on cylindrical functions $\mathscr{D} \subset \mathscr{H}_\omega$, and it can be extended in $\mathscr{H}_\omega$, to a self-adjoint (unbounded) operator (see ref. 3), which will be denoted later by the same symbol.

The operator (2) generates a single-spin dynamics with the flip rates

$$c(n, \sigma, \omega) = \frac{1}{1 + e^{-\Delta_n(\sigma, \omega)}},$$

$$\Delta_n(\sigma, \omega) = \beta H(\sigma^{(n)}, \omega) - \beta H(\sigma, \omega) = -2\beta(\omega_{n,n-1}\sigma_n\sigma_{n-1} + \omega_{n,n+1}\sigma_n\sigma_{n+1})$$

$\sigma^{(n)} \in \Omega$ is a configuration, which differs from the configuration $\sigma \in \Omega$ only at the point n:

$$\sigma_k^{(n)} = \begin{cases} \sigma_k, & k \neq n, \\ -\sigma_n, & k = n \end{cases}$$

We denote by

$$\sigma^\omega(t) = \{\sigma_n^\omega(t), t \geqslant 0, n \in \mathbf{Z}\}, \qquad \sigma^\omega(t) \in \Omega \tag{3}$$

the corresponding stationary Markov process on Ω with invariant measure $\mu_\beta(\omega)$, which is usually called as the Glauber dynamics. Let

$$S_\omega(t) = \exp\{tL^\beta(\omega)\}$$

denote the corresponding stochastic semigroup on $\mathscr{H}_\omega$.

In this paper basing on the spectral analysis of the generator (2) we obtain some new results on the relaxation to the equilibrium for the system (3) under additional assumptions on the random couplings.

We suppose, that

(1) the random variables ω_b are finite;

(2) the random variables ω_b are positive and bounded away from zero, so that:

$$0 < \gamma_0 \leqslant \omega_b \leqslant \gamma < \infty, \qquad \gamma_0 < \gamma, \qquad \gamma = \inf\{C\colon \Pr(\omega_b > C) = 0\} \tag{4}$$

(3) γ is the isolated maximum for ω_b, so that

$$\Pr(\omega_b = \gamma) = p_0 > 0, \quad \Pr(\gamma_1 < \omega_b < \gamma) = 0 \quad \text{with some } \gamma_0 < \gamma_1 < \gamma \tag{5}$$

Remark. The first condition guarantees the presence of the spectral gap of the generator (2) for any realization of the random couplings, see (16) below. The conditions (2)–(3) are necessary by technical reasons.

We denote by

$$L'(\omega) = L^\beta(\omega)|_{\mathscr{H}_\omega \ominus \{\mathbf{1}\}} \tag{6}$$

the restriction of the operator (2) on the space $\mathscr{H}_\omega \ominus \{\mathbf{1}\}$ of functions orthogonal to constants. The main result of the present paper is the calculation of the integrated density of states of the operator (6) near the upper spectrum edge. Then using the asymptotic formula for this function, we find the asymptotics for the averaged correlations:

$$\langle \langle \sigma_0^\omega(t), \sigma_0(0) \rangle_{\mathscr{P}(\omega)} \rangle_\omega \qquad \text{as} \quad t \to \infty$$

Here $\langle \cdot \rangle_{\mathscr{P}(\omega)}$ is the average over the process (3) corresponding to a fixed realization ω, and $\langle \cdot \rangle_\omega$ is the average over the distribution $\mathbf{P}$ of the random potential.

Our analysis is based on explicit representations for the restrictions of the generator (2) on invariant subspaces, which have been obtained in the paper.[2] Some partial results about invariant subspaces for this model without randomness have been known earlier, starting with the original paper of R. Glauber.[9] He observed in fact the first invariant subspace for the generator of the 1-D stochastic Ising model due to a simple structure of this subspace, see (8) below. There exist also results about the construction of invariant families of functions, using dual processes approach, see for instance the work of F. Spitzer.[10] However we need here more precise description of the invariant subspaces by the explicit construction of the orthonormal basis. So first we remind constructions and results about the complete spectral decomposition for the operator (2) from ref. 2. Let us note that results 1–3 below are valid for any fixed bounded realization $\omega \in \mathbf{R}^{\mathbf{B}}$.

1. For any bounded realization of the random couplings ω the space $\mathscr{H}_\omega$ is decomposed into a direct sum of subspaces invariant with respect to the operator $L^\beta(\omega)$:

$$\mathscr{H}_\omega = \overline{\bigoplus_{k=0}^{\infty} \mathscr{H}_\omega^{(k)}}$$

There is an orthonormal basis in $\mathscr{H}_\omega^{(k)}$ for any $k = 0, 1, 2\ldots$. Functions from this basis in $\mathscr{H}_\omega^{(k)}$ are marked by k-point subsets of the lattice

$$I = \{n_1, n_2, \ldots, n_k\} \subset \mathbf{Z}, \qquad |I| = k$$

and have a multiplicative structure:

$$v_I(\omega) = \prod_{n \in I} v_n(\omega), \qquad v_\varnothing \equiv 1$$

with

$$v_n(\omega) = D^{-1}(\omega)\, \sigma_n = \frac{\sigma_n - \tanh \beta\omega_{n-1,n} \cdot \sigma_{n-1}}{(1 - \tanh^2 \beta\omega_{n-1,n})^{1/2}}, \qquad n \in \mathbf{Z} \tag{7}$$

If we denote by

$$L_k^\beta(\omega) = L^\beta(\omega)|_{\mathscr{H}_\omega^{(k)}}$$

The restriction of $L^\beta(\omega)$ on the invariant subspace $\mathscr{H}_\omega^{(k)}$, then we have for the spectrum of $L^\beta(\omega)$:

$$\operatorname{spec} L^\beta(\omega) = \overline{\bigcup_{k=0}^{\infty} \operatorname{spec} L_k^\beta(\omega)}$$

2. For any bounded realization ω the linear span of the functions

$$\{\sigma_n, n \in \mathbf{Z}\} \tag{8}$$

is the same as the first invariant subspace $\mathscr{H}_{\omega}^{(1)}$, so that the functions (8) also form a basis (non-orthogonal) in $\mathscr{H}_{\omega}^{(1)}$. From (7) it follows that

$$\sigma_n = D(\omega)\, v_n(\omega) = \sum_{m \in \mathbf{Z}} D_{n,m}(\omega)\, v_m(\omega) \tag{9}$$

with

$$D_{n,n}(\omega) = (1 - \tanh^2 \beta\omega_{n-1,n})^{1/2},$$

$$D_{n,m}(\omega) = \tanh \beta\omega_{n-1,n} \ldots \tanh \beta\omega_{m,m+1}(1 - \tanh^2 \beta\omega_{m-1,m})^{1/2},$$
$$\text{if} \quad m < n$$

and

$$D_{n,m}(\omega) = 0, \qquad \text{if} \quad m > n$$

The operator $L_1(\omega) \equiv L_1^{\beta}(\omega)$ has the following representation in the basis (8):

$$L_1(\omega)\, \sigma_n = \mathscr{P}_n(\omega)\, \sigma_{n+1} - \sigma_n + \mathscr{B}_{n-1}(\omega)\, \sigma_{n-1} \tag{10}$$

Here

$$\mathscr{P}_n(\omega) = \tfrac{1}{2}(\tanh(\beta\omega_{n,n-1} + \beta\omega_{n,n+1}) - \tanh(\beta\omega_{n,n-1} - \beta\omega_{n,n+1})) \tag{11}$$

$$\mathscr{B}_{n-1}(\omega) = \tfrac{1}{2}(\tanh(\beta\omega_{n,n-1} + \beta\omega_{n,n+1}) + \tanh(\beta\omega_{n,n-1} - \beta\omega_{n,n+1})) \tag{12}$$

are bounded random fields.

3. For any bounded realization ω there is a unitary mapping:

$$V(\omega)\colon \mathscr{H}_{\omega}^{(1)} \to l_2(\mathbf{Z}), \qquad v_n(\omega) \to e_n, \qquad e_n(m) = \delta_{n,m} \in l_2(\mathbf{Z}) \tag{13}$$

and the operator $\tilde{L}_1(\omega) = V(\omega)\, L_1(\omega)\, V^*(\omega)$ has the following representation in the orthonormal basis $\{e_n, n \in \mathbf{Z}\}$ of the Hilbert space $l_2(\mathbf{Z})$:

$$\tilde{L}_1(\omega)\, e_n = \tilde{\mathscr{L}}_{n,n-1} e_{n-1} + \tilde{\mathscr{L}}_{n,n} e_n + \tilde{\mathscr{L}}_{n,n+1} e_{n+1} \tag{14}$$

by a symmetric matrix

$$\tilde{\mathscr{L}} = \tilde{\mathscr{L}}(\omega) = D^{-1}(\omega)\, \mathscr{L}(\omega)\, D(\omega) \tag{15}$$

where

$$\tilde{\mathscr{L}}_{n,n-1} = \frac{a_n(1-a_n^2)^{1/2}(1-a_{n-1}^2)^{1/2}}{(1-a_n^2 a_{n-1}^2)},$$

$$\tilde{\mathscr{L}}_{n,n+1} = \frac{a_{n+1}(1-a_n^2)^{1/2}(1-a_{n+1}^2)^{1/2}}{(1-a_n^2 a_{n+1}^2)},$$

$$\tilde{\mathscr{L}}_{n,n} = -1 - \frac{a_n^2(1-a_{n-1}^2)}{(1-a_n^2 a_{n-1}^2)} + \frac{a_{n+1}^2(1-a_n^2)}{(1-a_n^2 a_{n+1}^2)}$$

with

$$a_n = a_n(\omega) = \tanh \beta\omega_{n-1,n}$$

Here $\mathscr{L}(\omega)$ is the matrix, associated with the representation (10), and the matrices $D^{-1}(\omega)$, $D(\omega)$ are defined by (7) and (9).

4. In the case when random variables ω_b are uniformly bounded (i.e., under only the first assumption on the couplings):

$$|\omega_b| \leqslant \gamma, \qquad \gamma = \inf\{C>0\colon \Pr(|\omega_b|>C)=0\}$$

the spectrum of $L^\beta(\omega)$ is non-random for $\mathbf{P}$-a.e. ω, and it is the same as

$$\operatorname{spec} L^\beta(\omega) = \{0\} \cup \overline{\bigcup_{k=1}^{\infty} [-k-k\tanh 2\beta\gamma, -k+k\tanh 2\beta\gamma]}$$

where

$$\{0\} = \operatorname{spec} L_0^\beta(\omega),$$

$$[-k-k\tanh 2\beta\gamma, -k+k\tanh 2\beta\gamma] = \operatorname{spec} L_k^\beta(\omega), \qquad k\in\mathbf{N}$$

In this case for any fixed realization ω we have a spectral gap

$$g_\omega \geqslant 1 - \tanh 2\beta\gamma \tag{16}$$

and there is the spectral gap $g = 1-\tanh 2\beta\gamma$ with probability 1.

The main result of this paper are the following.

Theorem 1. Let ω_b, $b\in\mathbf{B}$, be i.i.d. random variables meeting (4)–(5) with $p_0 = \Pr(\omega_b=\gamma)$, where γ is the isolated maximum, and we denote by $N(L_1, d\lambda)$ the integrated density of states of the operator $L_1(\omega)$. Then

$$\ln N(L_1, (\lambda, \lambda_0)) = -\frac{\pi \sqrt{\tanh 2\beta\gamma}}{\sqrt{2(\lambda_0 - \lambda)}} \ln \frac{1}{p_0} (1 + o(1)), \qquad \text{as} \quad \lambda \nearrow \lambda_0 = -1 + \tanh 2\beta\gamma$$

Theorem 2. Under conditions of theorem 1 the following asymptotic formula for the time-autocorrelation function holds

$$\langle \langle \sigma_0^\omega(t), \sigma_0(0) \rangle_{\mathscr{P}(\omega)} \rangle_\omega = e^{-gt - kt^{1/3}(1 + o(1))} \qquad \text{as} \quad t \to \infty \tag{17}$$

with

$$g = 1 - \tanh 2\beta\gamma, \qquad k = k(p_0) = \frac{3}{2} \pi^{2/3} \left(\ln \frac{1}{p_0} \right)^{2/3} (\tanh 2\beta\gamma)^{1/3}$$

Remark 1. Let us note that Property 4 immediately implies the following upper bound on the correlations (uniformly by ω):

$$\langle \sigma_0^\omega(t), \sigma_0(0) \rangle_{\mathscr{P}(\omega)} \leqslant Ce^{-gt} \tag{18}$$

where $g = 1 - \tanh 2\beta\gamma$ and C is an absolute constant.

We will prove that the convergence to the equilibrium in average is more fast than the right-hand side of the estimate (18), and even more fast than in the translation-invariant case, when $\omega_b \equiv \gamma$ for any $b \in \mathbf{B}$:

$$\langle \sigma_0(t), \sigma_0(0) \rangle_{\mathscr{P}(\gamma)} = \frac{1}{\sqrt{t}} e^{-gt}(k_0 + o(1))$$

as $t \to \infty$ with a constant k_0. The last asymptotics has been proved in ref. 4.

Remark 2. In particular the asymptotics (17) holds, if the random variables ω_b take a finite number of positive values.

2. PROOF OF THEOREM 1

The proof is based on the representation (10) for the operator $L_1(\omega)$ and on methods for the calculation of the integrated density of states in some one-dimensional disordered systems, see refs. 5 and 6.

We introduce now the integrated density of states $N(L_1, d\lambda)$ for the random operator $L_1(\omega)$ by the following way. Let us consider truncations

$L_1^{(r)}(\omega)$, $r \in \mathbf{N}$ of the operator $L_1(\omega)$, corresponding to the representation (10). Namely the operators $L_1^{(r)}(\omega)$ is given by (10) in the finite-dimensional space of functions of the form

$$\left\{ f^{(r)}(\sigma) = \sum_{n=-r}^{r} f_n \sigma_n \right\}$$

Matrices $\mathscr{L}^{(r)}(\omega)$ associated to the operators $L_1^{(r)}(\omega)$ have the form of Jacobi matrices of the order $2r+1$ with positive entries $\mathscr{P}_n(\omega)$, $\mathscr{B}_n(\omega)$, $n = -r,..., r$. Consequently, for any r the operator $L_1^{(r)}(\omega)$ has only real eigenvalues, and the spectra of $L_1^{(r)}(\omega)$ are concentrated on the same real segment

$$\Delta = [-1 - \tanh 2\beta\gamma, \ -1 + \tanh 2\beta\gamma]$$

We denote by $k_r(L_1(\omega), \lambda)$ a non-decreasing function equal to the number of eigenvalues of the operator $L_1^{(r)}(\omega)$ not exceeding λ. Let $N_r(L_1(\omega), d\lambda)$ be a measure of $\mathbf{R}$, associated with the distribution function

$$N_r(L_1(\omega), \lambda) = N_r(L_1(\omega), (-\infty, \lambda)) = \frac{1}{2r+1} k_r(L_1(\omega), \lambda)$$

Lemma 1. There is a non-random positive measure $N(L_1, d\lambda)$ on $\mathbf{R}$, such that with probability one

$$\lim_{r \to \infty} N_r(L_1(\omega), d\lambda) = N(L_1, d\lambda) \tag{19}$$

in the sense of weak convergence of measures.

The measure $N(L_1, d\lambda)$ is relates to the resolution of the identity of the operator $\tilde{L}_1(\omega)$ by:

$$N(L_1, d\lambda) = \langle (E_{\tilde{L}_1(\omega)}(d\lambda))_{0,0} \rangle_\omega \equiv \langle (E_{\tilde{L}_1(\omega)}(d\lambda)\, e_0, e_0) \rangle_\omega \tag{20}$$

Proof. Follows by standard arguments (see, e.g., ref. 7). We prove that moments of the measures $N_r(L_1(\omega), d\lambda)$ converge. For any $p = 0$, 1, 2,... we have

$$\begin{aligned}(2r+1) \int_\Delta \lambda^p N_r(L_1(\omega), d\lambda) &= \mathrm{Tr}(L_1^{(r)}(\omega))^p \\ &= \sum_{\substack{|n| \leqslant r \\ |n_1| \leqslant r,..., |n_{p-1}| \leqslant r}} (\mathscr{L}^{(r)}(\omega))_{n, n_1} \cdots (\mathscr{L}^{(r)}(\omega))_{n_{p-1}, n}\end{aligned} \tag{21}$$

Here $\varDelta \subset \mathbf{R}$ is a segment, where the spectra of the operators $L_1^{(r)}(\omega)$ are concentrated for any r. As $r \to \infty$ we could rewrite the right-hand side of (21) as follows:

$$\begin{aligned}
&\sum_{\substack{|n| \leqslant r \\ n_1, \dots, n_{p-1} \in \mathbf{Z}}} (\mathscr{L}(\omega))_{n, n_1} \cdots (\mathscr{L}(\omega))_{n_{p-1}, n} + O(1) \\
&\quad = \sum_{|n| \leqslant r} (\mathscr{L}^p(\omega))_{n, n} + O(1) \\
&\quad = \sum_{|n| \leqslant r} (D^{-1}(\omega)\, \mathscr{L}^p(\omega)\, D(\omega))_{n, n} + O(1) \qquad (22)
\end{aligned}$$

where the matrices $D(\omega)$ and $D^{-1}(\omega)$ are defined by (9) and (7). Here $\mathscr{L}^p(\omega)$ and $D^{-1}(\omega)\, \mathscr{L}^p(\omega)\, D(\omega) = \widetilde{\mathscr{L}}^p(\omega)$ are infinite matrices associated with the representation of the same operator $L_1^p(\omega)$ in the bases

$$\{\sigma_n, n \in \mathbf{Z}\} \qquad \text{and} \qquad \{v_n, n \in \mathbf{Z}\}$$

respectively.

The last equality in (22) is the most important for us. The construction (13)–(15) implies that the right-hand side of (22) is the same as

$$\sum_{|n| \leqslant r} (\tilde{L}_1^p(\omega))_{n, n} + O(1) \qquad \text{as} \quad r \to \infty$$

From this and (21) applying the ergodic theorem, we have that with probability 1:

$$\begin{aligned}
\lim_{r \to \infty} \int_{\varDelta} \lambda^p N_r(L_1(\omega), d\lambda) &= \lim_{r \to \infty} \frac{1}{2r+1} \sum_{|n| \leqslant r} (\tilde{L}_1^p(\omega))_{n, n} \\
&= \langle (\tilde{L}_1^p(\omega))_{0, 0} \rangle_\omega \\
&= \int_{\varDelta} \lambda^p \langle (E_{\tilde{L}_1(\omega)}(d\lambda)\, e_0, e_0) \rangle_\omega \\
&= \int_{\varDelta} \lambda^p \langle (E_{\tilde{L}_1(\omega)}(d\lambda))_{0, 0} \rangle_\omega
\end{aligned}$$

Lemma 1 is proved.

3. CONSTRUCTION OF AN AUXILIARY OPERATOR $\hat{L}_1(\omega)$

We introduce now an operator $\hat{L}_1(\omega)$ in $l_2(\mathbf{Z})$ similar to $\tilde{L}_1(\omega)$, such that all components of the spectra of these operators (including pure point components) are the same. In addition the truncations $\hat{L}_1^{(r)}(\omega)$ and $L_1^{(r)}(\omega)$ of the operators $\hat{L}_1(\omega)$ and $L_1(\omega)$ have the same set of eigenvalues for any r.

We consider a random operator $\hat{L}_1(\omega)$ given in $l_2(\mathbf{Z})$ as follows:

$$(\hat{L}_1(\omega)\, f)(n) = \mathscr{P}_n(\omega)\, f(n+1) - f(n) + \mathscr{B}_{n-1}(\omega)\, f(n-1), \qquad f \in l_2(\mathbf{Z}) \tag{23}$$

with random fields $\mathscr{P}_n(\omega)$, $\mathscr{B}_n(\omega)$, $n \in \mathbf{Z}$, defined by (11)–(25). Then the following lemma holds.

Lemma 2. For any bounded realization of the random couplings ω the operators $\tilde{L}_1(\omega)$ and $\hat{L}_1(\omega)$ are similar, so that

(1) spec $\hat{L}_1(\omega) =$ spec $\tilde{L}_1(\omega) =$ spec $L_1(\omega)$,

(2) $\text{spec}_{pp}\, \hat{L}_1(\omega) = \text{spec}_{pp}\, \tilde{L}_1(\omega) = \text{spec}_{pp}\, L_1(\omega)$.

In addition for any truncations of the operators $\hat{L}_1(\omega)$ and $L_1(\omega)$ we have:

(3) spec $\hat{L}_1^{(r)}(\omega) =$ spec $L_1^{(r)}(\omega)$, and $k_r(\hat{L}_1(\omega), \lambda) = k_r(L_1(\omega), \lambda)$.

Proof. By (23) and (10) the operator $\hat{L}_1(\omega)$ has the following representation in the basis $\{e_n, n \in \mathbf{Z}\}$:

$$\hat{L}_1(\omega)\, e_n = \sum_m \mathscr{L}'_{n,m} e_m \tag{24}$$

where $\mathscr{L}'$ is the matrix transposed to the matrix $\mathscr{L}$, which has been introduced by (10) and (15). On the other hand, $\tilde{L}_1(\omega)$ is the self-adjoint operator in $l_2(\mathbf{Z})$, and by (14) and (15) we have:

$$\tilde{L}_1(\omega)\, e_n = \sum_m \tilde{\mathscr{L}}'_{n,m} e_m = \sum_m (D' \mathscr{L}' (D')^{-1})_{n,m}\, e_m \tag{25}$$

The representations (24) and (25) imply that the operators $\hat{L}_1(\omega)$ and $\tilde{L}_1(\omega)$ have the similar matrices in the basis $\{e_n\}$. Consequently, the operators $\hat{L}_1(\omega)$ and $\tilde{L}_1(\omega)$ are similar, and all components of their spectra are the same.

The last statement about truncated operators evidently follows from the representations (23) and (10), since the matrices corresponding to the operators $\hat{L}_1^{(r)}(\omega)$ and $L_1^{(r)}(\omega)$ are transposed. Lemma is proved.

We denote by

$$\hat{k}_r(L_1(\omega), \lambda) = (2r+1) - k_r(L_1(\omega), \lambda)$$

a function equal to the number of eigenvalues of $L_1^{(r)}(\omega)$ exceeding λ. Let us consider a random operator in $l_2(\mathbf{Z})$

$$A = A(\omega) = (2K-1)\,E - \hat{L}_1(\omega)$$

where $2K = \tanh 2\beta\gamma$; E is the identity operator, and denote as above by $A^{(r)}(\omega)$ the truncations of $A(\omega)$ on $[-r, r]$, $r \in \mathbf{N}$. Then Lemma 2 implies that

$$\operatorname{spec} A = [0, 2 \tanh 2\beta\gamma] \qquad \text{with probability 1}$$

and

$$k_r(A(\omega), a) - 1 \leqslant \hat{k}_r(L_1(\omega), \lambda) \leqslant k_r(A(\omega), a) \tag{26}$$

with $a = 2K - 1 - \lambda$ for any realization ω and any $r \in \mathbf{N}$.

4. THE OSCILLATION THEOREM

In this section we study spectral properties of the operator $A(\omega)$, corresponding some fixed realization ω of the random couplings. Namely, we will establish a connection between the shape of the realization ω and the number of eigenvalues of $A^{(r)}(\omega)$, which are near the zero. We use here the technique from ref. 5 based on the oscillation theorem. But we modify it to our case, when the system is given by random dependent fields.

We fix a large enough r and some realization ω. For any small positive $0 < a \ll 1$ we denote by

$$\alpha_{J(a)} = \alpha_{J(a)}^{(r)} = \max_{\alpha_j^{(r)} \leqslant a} \alpha_j^{(r)}$$

the maximal eigenvalue of $A^{(r)}(\omega)$ not exceeding a. Then

$$k_r(A(\omega), a) = J(a)$$

Let us consider the equation

$$(A^{(r)}(\omega)\, f)(n) = -\mathscr{P}_n(\omega)\, f(n+1) + 2Kf(n) - \mathscr{B}_{n-1}(\omega)\, f(n-1)$$
$$= \alpha_{J(a)} f(n) \tag{27}$$

on the interval $[-r, r]$ with some fixed boundary conditions. We notice that the matrix $\mathscr{A}_r(\omega)$ associated with the operator $A^{(r)}(\omega)$ (27) is a Jacobi matrix of the order $(2r+1)$, and $\mathscr{P}_n(\omega) > 0$, $\mathscr{B}_n(\omega) > 0$, $n \in \mathbf{Z}$. By the oscillation theorem for Jacobi matrices (see, e.g., ref. 8) there is exactly $(J(a)-1)$ alternations in sign in the sequence of coordinates $\{f_{J(a)}(n), n = -r,..., r\}$ of the eigenvector $f_{J(a)}(n)$ for $\mathscr{A}_r(\omega)$ that is the solution of (27). It is suitable in the general case to introduce the standard phase $\varphi_j(n)$ as

$$\cot \varphi_j(n) = \frac{f_j(n)}{f_j(n-1)}, \qquad n = -r+1,..., r$$

where $\{f_j(n)\}_{n=-r}^{r}$ is the eigenvector of $\mathscr{A}_r(\omega)$, corresponding to the eigenvalue α_j. Therefore if we denote by $m_r(\varphi_{J(a)})$ the number of such points $k \in [-r, r]$, where

$$\cot \varphi_{J(a)}(k) < 0$$

then

$$k_r(A(\omega), a) = m_r(\varphi_{J(a)}) + 1 \tag{28}$$

Let us preserve the notation $f(n)$ for the solution of (27) equal to the eigenvector $f_{J(a)}(n)$, and $\varphi(n)$ for the corresponding phase $\varphi_{J(a)}(n)$. We rewrite (27) as

$$\begin{pmatrix} f(n+1) \\ f(n) \end{pmatrix} = \begin{pmatrix} \dfrac{2K - \alpha_{J(a)}}{\mathscr{P}_n(\omega)} & -\dfrac{\mathscr{B}_{n-1}(\omega)}{\mathscr{P}_n(\omega)} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f(n) \\ f(n-1) \end{pmatrix}$$

and denote by $T_n = T_n(\omega, a)$ the transition matrix at point n:

$$\begin{pmatrix} f(n+1) \\ f(n) \end{pmatrix} = T_n \begin{pmatrix} f(n) \\ f(n-1) \end{pmatrix}$$

In the case when a point n connects random variables taking maximal value γ:

$$\omega_{n-1,n} = \omega_{n,n+1} = \gamma$$

the transition matrix T^0 has a form

$$T^0 = \begin{pmatrix} 2 - \dfrac{\alpha_{j(a)}}{K} & -1 \\ 1 & 0 \end{pmatrix}$$

By a transformation

$$S = \begin{pmatrix} -e^{i\vartheta} & e^{-i\vartheta} \\ -1 & 1 \end{pmatrix} \tag{29}$$

one can diagonalize T^0:

$$S^{-1}T^0S = \begin{pmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{pmatrix} \tag{30}$$

with

$$\cos\vartheta = 1 - \frac{\alpha_{J(a)}}{2K}, \qquad 0 < \vartheta < \pi$$

We recall that $\alpha_{J(a)}$ is small enough: $0 < \alpha_{J(a)} \leqslant a \ll 1$. Consequently, $\vartheta = \vartheta(a)$ is also small, and has the order

$$\vartheta \sim \frac{\sqrt{\alpha_{J(a)}}}{\sqrt{K}}$$

We introduce now a new phase $\chi(n)$ by

$$S^{-1}\begin{pmatrix} \cos\varphi(n) \\ \sin\varphi(n) \end{pmatrix} = C_n \begin{pmatrix} e^{i\chi(n)} \\ e^{-i\chi(n)} \end{pmatrix}, \qquad n = -r+1,..., r \tag{31}$$

with the standard phase $\varphi(n) = \varphi_{J(a)}(n)$ and constants C_n. From (29) and (31) it follows that

$$\cot\varphi(n) = \sin\vartheta \cdot \cot\chi(n) + \cos\vartheta \tag{32}$$

Remark. The relation (32) implies, that $\cot\varphi(k) < 0$ if and only if

$$\cot\chi(k) < -\cot\vartheta, \qquad \text{i.e.,} \quad \chi(k) \in (\pi l - \vartheta, \pi l] \tag{33}$$

with some $l \in \mathbf{N}$. Thus if we denote by $m_r(\chi)$ the number of such points $k \in [-r, r]$, where

$$\cot \chi(k) < -\cot \vartheta$$

then it follows from (28) that

$$k_r(A(\omega), a) = m_r(\chi_{J(a)}) + 1 \tag{34}$$

Let us consider the behavior $\chi(n)$ in two cases.

1. So-called regular case, when $\omega_{n-1,n} = \omega_{n,n+1} = \gamma$. Then $T_n = T^0$, and (30), (31) immediately imply that

$$\cot \chi(n+1) = \cot(\chi(n) + \vartheta) \tag{35}$$

so that the phase $\chi(n)$ increases linearly on the parts of the interval $[-r, r]$, where the couplings take the maximal value.

2. In the general case, when at least one of $\omega_{n-1,n}$, $\omega_{n,n+1}$ does not equal to γ, $T_n \neq T^0$, and we can rewrite T_n as

$$T_n = T^0 + \delta_n$$

with

$$\delta_n = \begin{pmatrix} \dfrac{(2K - \alpha_{J(a)})(K - \mathscr{P}_n)}{K\mathscr{P}_n} & 1 - \dfrac{\mathscr{B}_{n-1}}{\mathscr{P}_n} \\ 0 & 0 \end{pmatrix} \tag{36}$$

By (31), (29), (30) and (36) we get the following relation

$$\begin{aligned} \cot \chi(n+1) &= \frac{\mathscr{B}_{n-1}}{\mathscr{P}_n} \cot(\chi(n) + \vartheta) + \frac{1}{\sin \vartheta} \frac{(2K - \alpha_{J(a)})(K - \mathscr{P}_n)}{K\mathscr{P}_n} \\ &\quad + \left(1 - \frac{\mathscr{B}_{n-1}}{\mathscr{P}_n}\right) \cot \vartheta \\ &= \frac{\mathscr{B}_{n-1}}{\mathscr{P}_n} \cot(\chi(n) + \vartheta) + \frac{2K - \mathscr{P}_n - \mathscr{B}_{n-1}}{\mathscr{P}_n} \cot \vartheta \end{aligned} \tag{37}$$

The conditions (4)–(5) on the couplings imply that in this case

$$0 < \frac{1}{W} \leqslant \Phi_n(\omega) \equiv \frac{\mathscr{B}_{n-1}(\omega)}{\mathscr{P}_n(\omega)} \leqslant W < \infty \tag{38}$$

and

$$\infty > V_n(\omega) \equiv \frac{2K - \mathscr{P}_n(\omega) - \mathscr{B}_{n-1}(\omega)}{\mathscr{P}_n(\omega)} \geqslant \delta > 0 \tag{39}$$

with

$$W \equiv W(\gamma_0, \gamma) = \frac{\tanh(\beta\gamma + \beta\gamma_0) + \tanh(\beta\gamma - \beta\gamma_0)}{\tanh(\beta\gamma + \beta\gamma_0) - \tanh(\beta\gamma - \beta\gamma_0)} > 1, \tag{40}$$

$$0 < \delta \equiv \delta(\gamma_0, \gamma, \gamma_1) < 1 \tag{41}$$

The bounds (38)–(39) are uniform by the random couplings ω and depend on parameters of the distribution p of the potential ω_b: γ_0, γ and γ_1, where $\gamma_1 = \sup(\operatorname{supp} p \backslash \gamma) < \gamma$. Now we could rewrite the equation (37) using the notations from (38)–(39) by the following way

$$\cot \chi(n+1) = \Phi_n(\omega) \cot(\chi(n) + \vartheta) + V_n(\omega) \cot \vartheta \tag{42}$$

Definition. We call an interval $[k_1, k_2] \subseteq [-r, r]$ as *regular*, if $\omega_b = \gamma$ for any bond $b \in [k_1, k_2]$ of this interval.

Lemma 3 (The "invariance" property for χ). Let ϑ be small enough. If

$$0 < \chi(k) < \chi_{cr} = \operatorname{arccot} \frac{\delta}{2W\vartheta}$$

and the point $(k-1)$ is the beginning of a regular interval of the length s with

$$0 < s < s_{cr} = \frac{\pi}{\vartheta} - \frac{4W}{\delta}$$

then

$$0 < \chi(k+s) < \chi_{cr}$$

Furthermore, for any point in the regular interval we have

$$\chi(k+s') < \pi - 3\vartheta, \qquad s' < s$$

so that $\chi(k+s')$ doesn't meet (33), when $s' \leqslant s < s_{cr}$.

Proof. Follows from (35) and (42) in a similar way as in ref. 5. The relation (35) together with the inequalities $\operatorname{arccot} x < 1/x$ $(x > 0)$ and (40)–(41) imply that

$$\chi(k+s-1) = \chi(k) + (s-1)\,\vartheta$$
$$< \frac{2W\vartheta}{\delta} + \pi - \frac{4W\vartheta}{\delta} - \vartheta = \pi - \frac{2W\vartheta}{\delta} - \vartheta < \pi - 3\vartheta$$

This bound is also valid for any positive integer $s' \leqslant s < s_{cr}$. Since $\omega_{k+s-1,\,k+s} < \gamma$, then by (38)–(42) we get

$$\cot \chi(k+s) > -W \cot \frac{2W\vartheta}{\delta} + \frac{\delta}{\vartheta} - O(\vartheta) >$$
$$> \frac{\delta}{2\vartheta} - O(\vartheta) > \frac{\delta}{2W\vartheta} = \cot \chi_{cr} \tag{43}$$

Here we used the inequality $\cot x < 1/x$ and the inequality $\cot x > 1/x - O(x)$, which is valid for small positive x. Since $\cot x$ is the monotone decreasing function, the inequality (43) implies that

$$0 < \chi(k+s) < \chi_{cr}$$

Lemma is proved.

Lemma 3 shows that between two consecutive alternations in sign in the coordinates for $f_{J(a)}(n)$ one must find at least one regular interval of the length $s \geqslant s_{cr}$. On the other hand by (35) any regular interval of the length greater than $[\pi/\vartheta]$ provides an additional alternation in sign for $f_{J(a)}(n)$. From this reasoning and (34) we get the following inequality:

$$\mathcal{R}_r\left(\left[\frac{\pi}{\vartheta}\right]+1\right)+1 \leqslant k_r(A(\omega), a) \leqslant \mathcal{R}_r(s_{cr})+2 \tag{44}$$

where $\mathcal{R}_r(s)$ is the number of intervals of the length s, which can be arranged without overlapping within the regular intervals on $[-r, r]$ associated with the realization ω on $[-r, r]$. From (44) and (26) it follows that

$$\frac{1}{2r+1}\,\mathcal{R}_r\left(\left[\frac{\pi}{\vartheta}\right]+1\right) \leqslant \frac{\hat{k}_r(L_1(\omega), \lambda)}{2r+1} \leqslant \frac{1}{2r+1}\,\mathcal{R}_r(s_{cr}) + \frac{2}{2r+1} \tag{45}$$

with $\lambda = \lambda_0 - a$, where $\lambda_0 = -1 + \tanh 2\beta\gamma$ is the upper edge of the a.e.-spectrum of $L_1(\omega)$.

By averaging the inequality (45) over all realizations, after taking the limit $r \to \infty$ we have:

$$c \sum_{j=1}^{\infty} P\left(j\left(\left[\frac{\pi}{\vartheta}\right]+1\right)\right) \leqslant \hat{N}(L_1, \lambda) \leqslant c \sum_{j=1}^{\infty} P(js_{cr}) \tag{46}$$

where

$$\hat{N}(L_1, \lambda) = \int_{\lambda}^{\lambda_0} N(L_1, d\lambda) = N(L_1, (\lambda, \lambda_0))$$

and the measure $N(L_1, d\lambda)$ was defined in (19);

$$s_{cr} = \frac{\pi}{\vartheta} - d, \qquad d = \frac{4W}{\delta} \tag{47}$$

is the absolute constant,

$$\vartheta = \frac{\sqrt{\lambda_0 - \lambda}}{\sqrt{K}} (1 + \varepsilon_0(\lambda)), \qquad \varepsilon_0(\lambda) \to 0 \qquad \text{as} \quad \lambda \to \lambda_0 - 0 \tag{48}$$

and

$$P(js) = \frac{p_0^{js}}{1 - p_0} \tag{49}$$

is the probability that the regular interval has a length nog less than js. Here we used lemma 1 and the ergodicity of the random field ω, which implies that

$$\lim_{r \to \infty} \frac{1}{2r+1} \langle \mathcal{R}_r(s) \rangle_{\omega} = c \sum_{j=1}^{\infty} P(js)$$

with $c = p_0(1 - p_0)$. Taking into account (47)–(49), we could rewrite (46) as:

$$-\frac{\pi}{\vartheta} \ln \frac{1}{p_0} + k_1 \leqslant \ln \hat{N}(L_1, \lambda) \leqslant -\frac{\pi}{\vartheta} \ln \frac{1}{p_0} + k_2$$

with constants $k_j = k_j(p_0, W, \delta)$, $j = 1, 2$, $k_1 < k_2$.

Therefore the following, asymptotic formula holds

$$\ln \hat{N}(L_1, \lambda) = -\frac{\pi\sqrt{K}}{\sqrt{\lambda_0-\lambda}} \ln \frac{1}{p_0}\,(1+\varepsilon(\lambda)), \qquad \varepsilon(\lambda)\to 0 \quad \text{as} \quad \lambda \nearrow \lambda_0 \tag{50}$$

Theorem 1 is proved.

5. PROOF OF THEOREM 2

In what follows we shall use the notation $\langle\cdot\rangle \equiv \langle\cdot\rangle_\omega$ for the average over $\mathbf{P}$. Using the spectral theorem together with properties 1–3 of the generator $L^\beta(\omega)$ we have:

$$\begin{aligned}\langle\langle\sigma_0^\omega(t), \sigma_0(0)\rangle_{\mathscr{P}(\omega)}\rangle &= \left\langle \int e^{t\lambda}(E_{L^\beta(\omega)}(d\lambda)\,\sigma_0, \sigma_0)_{\mathscr{H}_\omega}\right\rangle \\ &= \int e^{t\lambda}\langle(E_{L_1(\omega)}(d\lambda)\,\sigma_0, \sigma_0)_{\mathscr{H}_\omega^{(1)}}\rangle \\ &= \int e^{t\lambda}\langle(E_{\tilde{L}_1(\omega)}(d\lambda)\,f_0, f_0)_{l_2(\mathbf{Z})}\rangle \end{aligned} \tag{51}$$

Here

$$\{E_{L^\beta(\omega)}(d\lambda)\},\ \{E_{L_1(\omega)}(d\lambda)\},\ \{E_{\tilde{L}_1(\omega)}(d\lambda)\}$$

are the resolutions of the identity of the operators $L^\beta(\omega)$, $L_1(\omega)$, $\tilde{L}_1(\omega)$ respectively, and

$$f_0 = f_0(\omega) = V(\omega)\,\sigma_0 \in l_2(\mathbf{Z})$$

We denote by $(E_{\tilde{L}_1(\omega)}(d\lambda))_{n,m}$ the matrix elements of $E_{\tilde{L}_1(\omega)}(d\lambda)$ in the orthonormal basis $\{e_n\}$ of $l_2(\mathbf{Z})$:

$$(E_{\tilde{L}_1(\omega)}(d\lambda))_{n,m} = (E_{\tilde{L}_1(\omega)}(d\lambda)\,e_n, e_m)$$

Lemma 4. There exist such absolute constants $0 < C_1 < C_2 < \infty$, that

$$C_1\langle(E_{\tilde{L}_1(\omega)}(d\lambda))_{0,0}\rangle \leqslant \langle(E_{\tilde{L}_1(\omega)}(d\lambda)\,f_0, f_0)\rangle \leqslant C_2\langle(E_{\tilde{L}_1(\omega)}(d\lambda))_{0,0}\rangle \tag{52}$$

Proof. Under assumption (4) we have the following for the coefficients $D_{n,m}(\omega)$ in the decomposition (9):

$$D_{n,m}(\omega) > 0, \qquad \text{if} \quad m \leqslant n \tag{53}$$

$$\sup_{\omega} \sup_{n} \sum_{m} D_{n,m}(\omega) \equiv d(\gamma) < \infty \tag{54}$$

with a constant $d(\gamma)$. Using (53)–(54) we get the upper bound:

$$\begin{aligned}
\langle (E_{\tilde{L}_1(\omega)}(d\lambda)\, f_0, f_0)\rangle &= \langle (E_{\tilde{L}_1(\omega)}(d\lambda)\, V(\omega)\, D(\omega)\, v_0, V(\omega)\, D(\omega)\, v_0)\rangle \\
&= \left\langle \left(\sum_n D_{0,n}(\omega)\, E_{\tilde{L}_1(\omega)}(d\lambda)\right) e_n, \sum_m D_{0,m}(\omega)\, e_m \right)\right\rangle \\
&= \left\langle \sum_{n,m} D_{0,n}(\omega)\, D_{0,m}(\omega) (E_{\tilde{L}_1(\omega)}(d\lambda))_{n,m} \right\rangle \\
&\leqslant d^2(\gamma) \sup_{m,n} \langle (E_{\tilde{L}_1(\omega)}(d\lambda))_{n,m}\rangle \\
&\leqslant d^2(\gamma) \langle (E_{\tilde{L}_1(\omega)}(d\lambda))_{0,0}\rangle
\end{aligned} \tag{55}$$

In the last estimate we used Cauchy–Schwarz–Bunyakovskii inequality and the ergodicity of $\tilde{L}_1(\omega)$.

We denote by

$$c = \max_{\omega} \frac{1}{1 - \tanh^2 \beta\omega_{-1,0}} = \frac{1}{1 - \tanh^2 \beta\gamma}$$

Then the lower bound follows from (7) and (13):

$$\begin{aligned}
\langle (E_{\tilde{L}_1(\omega)}(d\lambda))_{0,0}\rangle &= \langle (E_{L_1(\omega)}(d\lambda)\, v_0, v_0)\rangle \\
&\leqslant c \cdot \langle (E_{L_1(\omega)}(d\lambda)\, \sigma_0, \sigma_0) \\
&\quad + 2 \tanh \beta\omega_{-1,0}\, |(E_{L_1(\omega)}(d\lambda)\, \sigma_0, \sigma_{-1})| \\
&\quad + \tanh^2 \beta\omega_{-1,0} \cdot (E_{L_1(\omega)}(d\lambda)\, \sigma_{-1}, \sigma_{-1})\rangle \\
&\leqslant 4c \langle (E_{L_1(\omega)}(d\lambda)\, \sigma_0, \sigma_0)\rangle = 4c \langle (E_{\tilde{L}_1(\omega)}(d\lambda)\, f_0, f_0)\rangle
\end{aligned}$$

We used here also Cauchy–Schwarz–Bunyakovskii inequality and the ergodicity of $L_1(\omega)$.

Setting

$$C_1 = \frac{1}{4c}, \qquad C_2 = d^2(\gamma)$$

we get (52). Lemma is proved.

Finally by the integration by parts from (51), (52), (20) and (50) we have for $I(t) = \langle \langle \sigma_0^\omega(t), \sigma_0(0) \rangle_{\mathscr{P}(\omega)} \rangle_\omega$ as $t \to \infty$

$$I(t) \leqslant C_2 \left[te^{t\lambda_0} \int_0^a \exp\left\{ -t\mu - \frac{\pi \sqrt{K}}{\sqrt{\mu}} \ln \frac{1}{p_0} (1 + \varepsilon(\mu)) \right\} d\mu + O(e^{t(\lambda_0 - a)}) \right] \tag{56}$$

$$I(t) \geqslant C_1 \left[te^{t\lambda_0} \int_0^a \exp\left\{ -t\mu - \frac{\pi \sqrt{K}}{\sqrt{\mu}} \ln \frac{1}{p_0} (1 + \varepsilon(\mu)) \right\} d\mu + O(e^{t(\lambda_0 - a)}) \right] \tag{57}$$

Here C_1, C_2 are constants, $\lambda_0 = -1 + \tanh 2\beta\gamma$, $K = \frac{1}{2} \tanh 2\beta\gamma$, $\mu = \lambda_0 - \lambda$, $\varepsilon(\mu) \to 0$ as $\mu \to 0$, and a is small enough. After changing of variables $\mu = t^{-2/3} u$ we could rewrite the first term in the right-hand side of (56)–(57) as:

$$\begin{aligned} &Cte^{t\lambda_0} \int_0^\infty \exp\left\{ -t^{1/3} \left(u + \frac{\pi \sqrt{K}}{\sqrt{u}} \ln \frac{1}{p_0} \left(1 + \varepsilon\left(\frac{u}{t^{2/3}} \right) \right) \right) \right\} du \\ &\qquad = \exp\left\{ -tg - t^{1/3} \left(\ln \frac{1}{p_0} \right)^{2/3} (\tanh 2\beta\gamma)^{1/3} \left[\frac{3}{2} \pi^{2/3} + o(1) \right] \right\} \end{aligned}$$

as $t \to \infty$, $g = |\lambda_0| = 1 - \tanh 2\beta\gamma$. This formula together with (56)–(57) implies the asymptotics (17).

Theorem 2 is proved.

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